# Solving Differential Equations with MATLAB

Hans-Petter Halvorsen

# Solving Differential Equations in MATLAB

MATLAB have lots of built-in functionality for solving differential equations. MATLAB includes functions that solve ordinary differential equations (ODE) of the form:

$$\frac{dy}{dt} = f(t, y), \qquad y(t_0) = y_0$$

MATLAB can solve these equations numerically.

Higher order differential equations must be reformulated into a system of first order differential equations.

**Note!** Different notation is used:

$$\frac{dy}{dt} = y' = \dot{y}$$

Not all differential equations can be solved by the same technique, so MATLAB offers lots of different ODE solvers for solving differential equations, such as **ode45**, **ode23**, **ode113**, etc.

# Bacteria Population

In this task we will simulate a simple model of a bacteria population in a jar.

The model is as follows:

$$birth\ rate = bx$$

$$death\ rate = px^2$$

Then the total rate of change of bacteria population is:

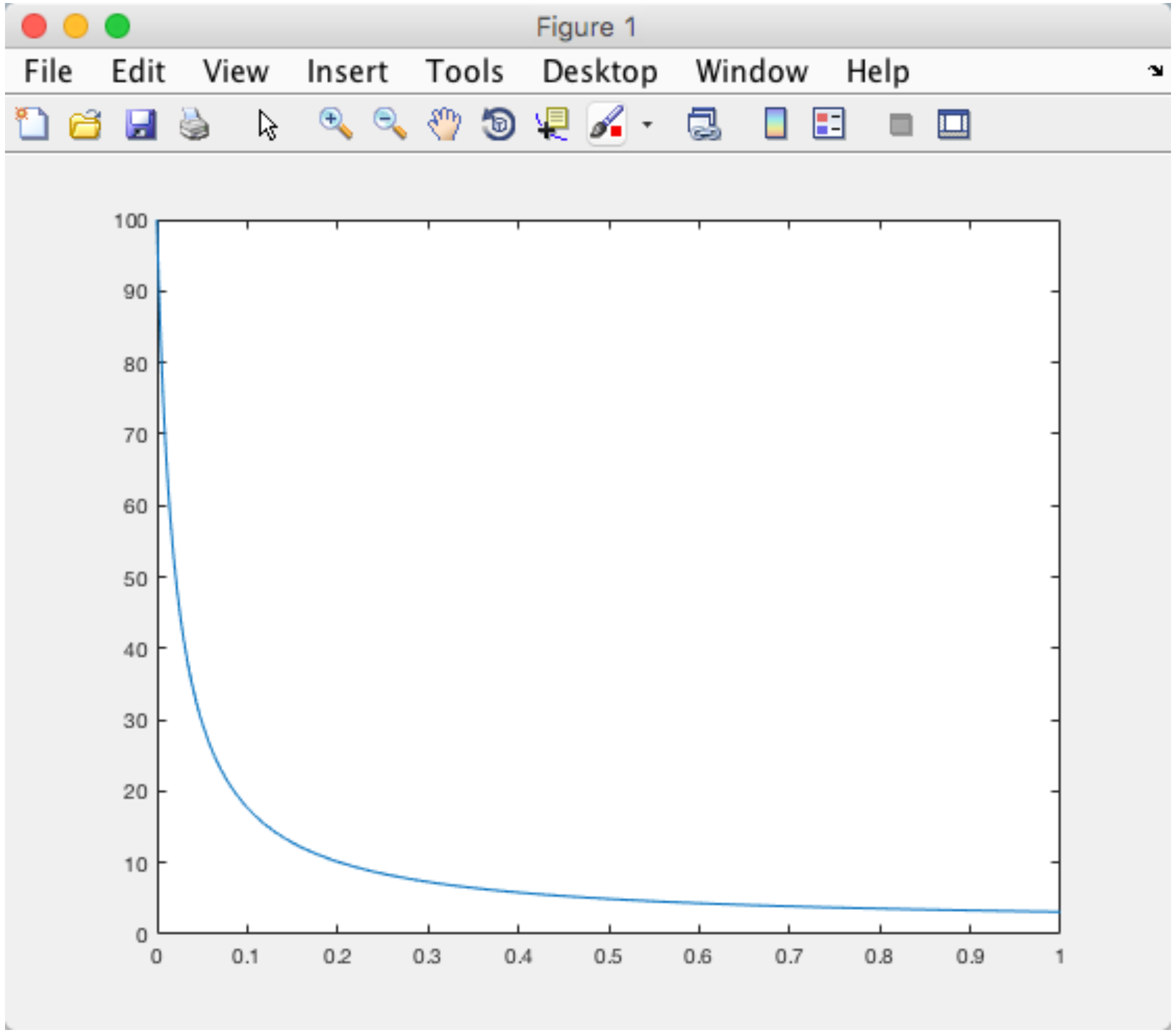
$$\dot{x} = bx - px^2$$

Set ***b=1***/hour and ***p=0.5*** bacteria-hour

→ Simulate the number of bacteria in the jar after **1 hour**, assuming that initially there are **100 bacteria** present.

```
function dx = bacteriadiff(t,x)
% My Simple Differential Equation

b=1;
p=0.5;

dx = b*x - p*x^2;
```

```
clear
clc

tspan=[0 1];
x0=100;

[t,y]=ode45(@bacteriadiff, tspan,x0);
plot(t,y)

[t,y]
```



# Passing Parameters to the model

Given the following system (1.order differential equation):

$$\dot{x} = ax + b$$

where $a = -\frac{1}{T}$ ,where $T$ is the time constant

In this case we want to pass $a$ and $b$ as parameters, to make it easy to be able to change values for these parameters

We set $b = 1$

We set initial condition $x(0) = 1$ and $T = 5$.

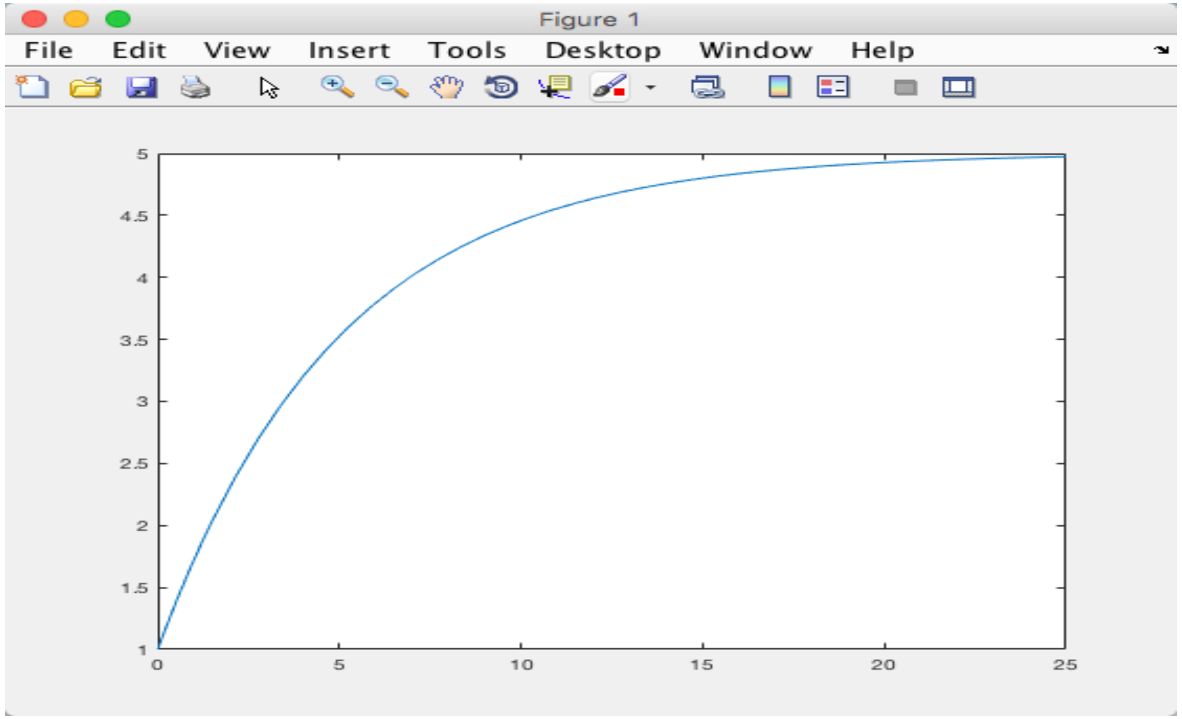
Solve the Equation and Plot the results with MATLAB

```
function dx = mysimplediff(t,x,param)
% My Simple Differential Equation

a=param(1);
b=param(2);

dx=a*x+b;
```

By doing this, it is very easy to changes values for the parameters a and b.

**Note!** We need to use the 5. argument in the ODE solver function for this. The 4. argument is for special options and is normally set to “[]”, i.e., no options.

```
tspan=[0 25];
x0=1;
a=-1/5;
b=1;
param=[a b];

[t,y]=ode45(@mysimplediff, tspan,
x0,[], param);
plot(t,y)
```



# Differential Equation

Use the ode23 function to solve and plot the results of the following differential equation in the interval $[t_0, t_f]$:

$$w' + (1.2 + sin10t)w = 0$$

*Where:*

$$t_0 = 0$$
$$t_f = 5$$
$$w(t_0) = 1$$

# Differential Equation

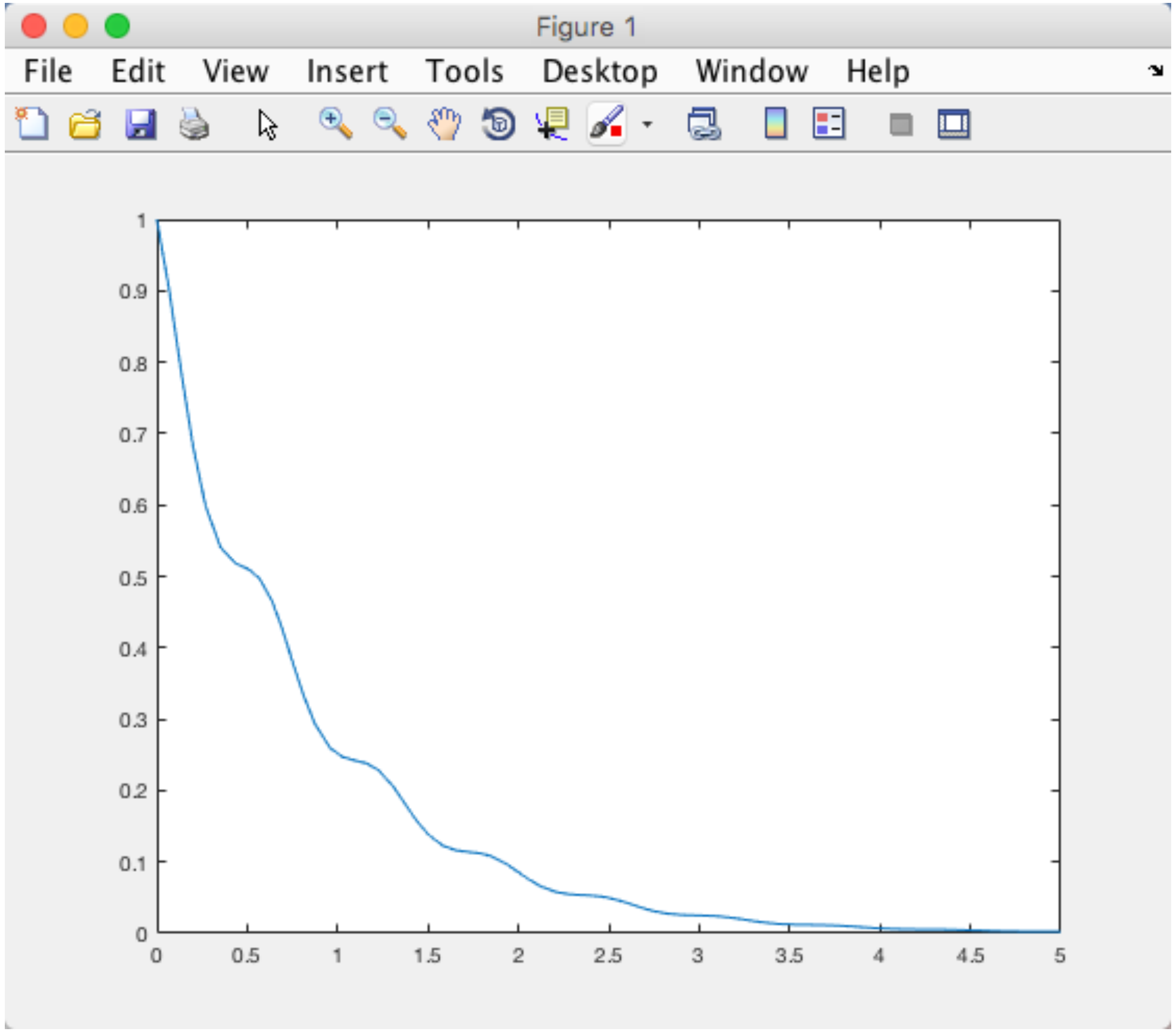
We start by rewriting the differential equation:

$$w' = -(1.2 + sin10t)w$$

Then we can implement it in MATLAB

```
function dw = diff_task3(t,w)

dw = -(1.2 + sin(10*t))*w;
```

```
tspan=[0 5];
w0=1;

[t,w]=ode23(@diff_task3, tspan, w0);
plot(t,w)
```



# 2.order differential equation

Use the ode23/ode45 function to solve and plot the results of the following differential equation in the interval $[t_0, t_f]$:

$$(1 + t^2)\ddot{w} + 2t\dot{w} + 3w = 2$$

Where; , $t_0 = 0, t_f = 5, w(t_0) = 0, \dot{w}(t_0) = 1$

**Note!** Higher order differential equations must be reformulated into a system of first order differential equations.

**Tip 1:** Reformulate the differential equation so $\ddot{w}$ is alone on the left side.

**Tip 2:** Set:

$$w = x_1$$
$$\dot{w} = x_2$$

# 2.order differential equation

Tip1: First we rewrite like this:

$$\ddot{w} = \frac{2 - 2t\dot{w} - 3w}{(1 + t^2)}$$

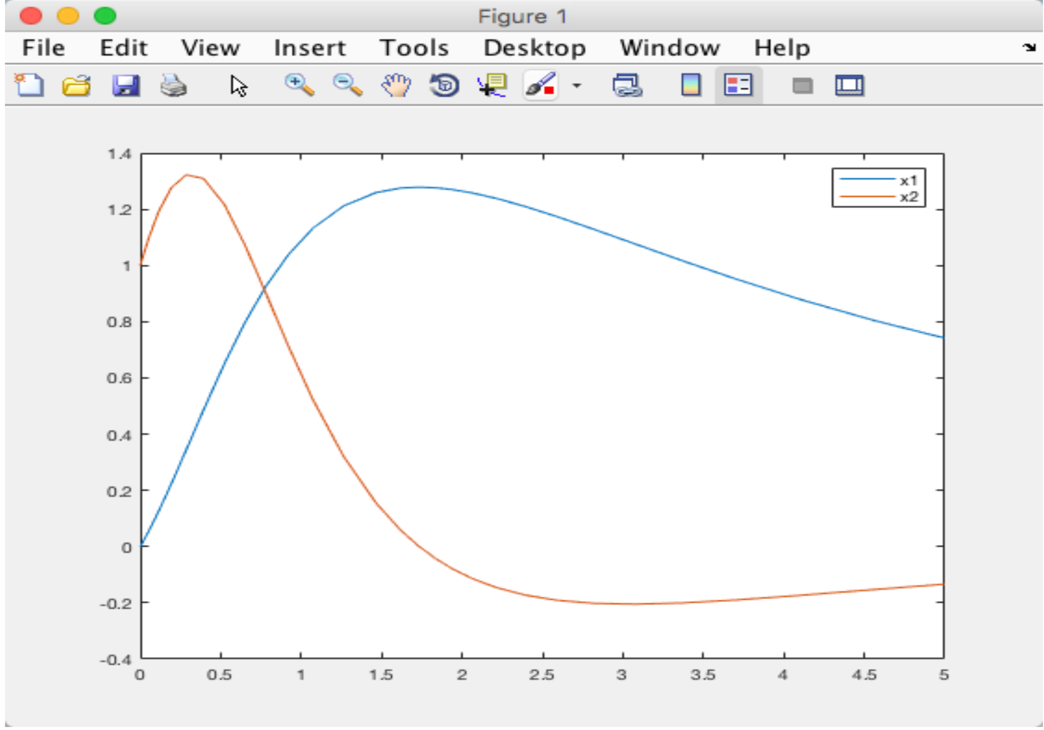
Tip2: In order to solve it using the ode functions in MATLAB it has to be a set with 1.order ode's. So we set:

$$w = x_1$$
$$\dot{w} = x_2$$

This gives 2 first order differential equations:

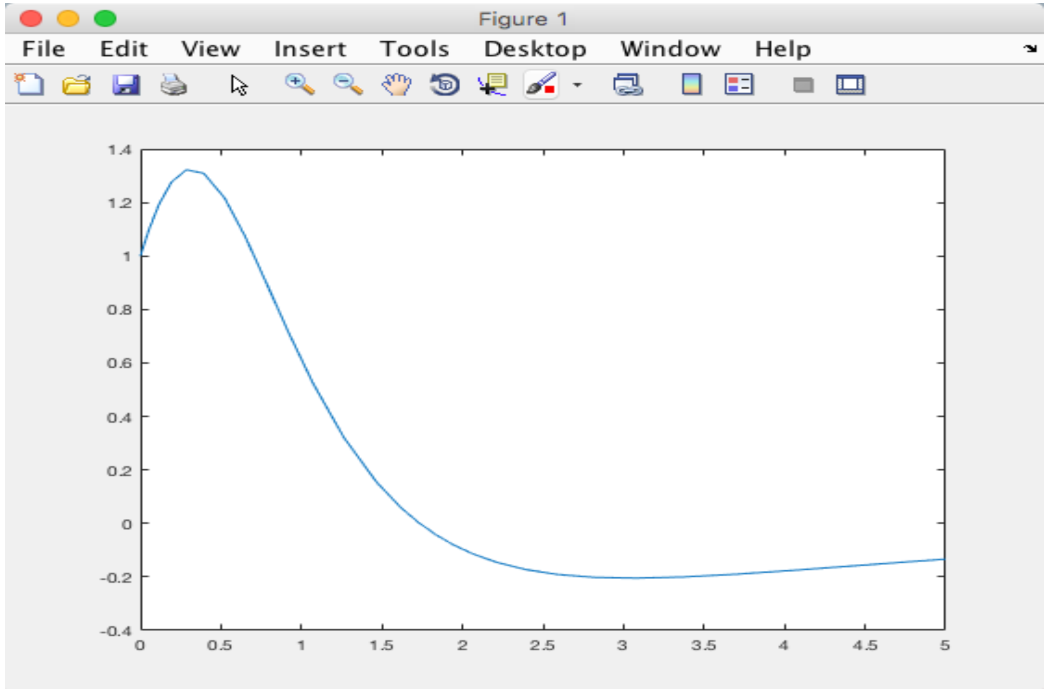
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{2 - 2tx_2 - 3x_1}{(1 + t^2)}$$

```
function dx = diff_secondorder(t,x)

[m,n] = size(x);
dx = zeros(m,n)

dx(1) = x(2);
dx(2) = (2-2*t*x(2)-3*x(1))/(1+t^2);
```




```
tspan=[0 5];
x0=[0; 1];

[t,x]=ode23(@diff_secondorder, tspan, x0);
plot(t,x)
legend('x1','x2')
```

```
tspan=[0 5];
x0=[0; 1];

[t,x]=ode23(@diff_secondorder, tspan, x0);
plot(t, x(:,2))
```



# Hans-Petter Halvorsen

University of South-Eastern Norway

www.usn.no

E-mail: hans.p.halvorsen@usn.no

Web: https://www.halvorsen.blog